MATCHING AS A TOOL TO DECOMPOSE WAGE GAPS

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Abstract—This paper presents a methodology that uses matching comparisons to explain gender wage differences. The approach emphasizes gender differences in the supports of the distributions of observable characteristics and provides insights into the distribution of unexplained gender pay differences. This nonparametric alternative to the Blinder-Oaxaca (BO) decomposition does not require the estimation of earnings equations and divides the gap into four additive elements. Two of these are analogous to the elements of the BO decomposition (but computed only over the common support of the distributions of characteristics), while the other two account for differences in the supports.

I. Introduction

Gender differences in the labor market, particularly the gender wage gap, have been an area of concern for theoretical and empirical research. On average, males earn more than females, yearly, monthly, and per hour. These differences in average earnings—the gender gaps—are partially explained by gender differences in individuals’ observable characteristics that the labor market rewards. The wage gap decomposition developed by Blinder and Oaxaca in 1973 has been a key tool in explaining the wage gap and the role that differences in individual characteristics play. This decomposition requires the linear regression estimation of earnings equations for both females and males. Based on these earnings equations, it generates the counterfactual: “What would a male earn if the compensation scheme for his individual characteristics aligned with that of a female?” Based on that counterfactual, the difference in average wages between males and females is broken into two additive components: one attributable to differences in the average characteristics of the individuals, and the other to differences in the average rewards that these characteristics have. The latter component is considered to contain the effects of both unobservable gender differences in characteristics and discrimination in the labor market.

There is a potential problem associated with this approach: misspecification due to differences in the supports of the empirical distributions of individual characteristics for females and males (hereafter called gender differences in the supports). While this is a problem largely recognized in the program evaluation literature, it has not received attention in the analysis of wage gaps.1 There are combinations of individual characteristics for which it is possible to find males in the labor force, but not females. For example, males who are in their early thirties, married, and hold at least a college degree. There are also combinations of characteristics for which it is possible to find females, but not males. For example, single females that are migrants, in their late forties, and have less than an elementary school education. With such combinations of characteristics, one cannot compare wages across genders. This problem of comparability is accentuated when job characteristics are included in the explanation of the wage gap. While females tend to concentrate in certain occupations that demand low-risk manual skills (for example, nurses or maids), males are more likely to be found working in hazardous or managerial occupations that require long tenure.2

The traditional Blinder-Oaxaca (BO) decomposition fails to recognize these gender differences in the supports by estimating earnings equations for all working females and males without restricting the comparison only to those individuals with comparable characteristics. By not considering this restriction, the BO decomposition is implicitly based on an “out-of-support assumption”: it becomes necessary to assume that the linear estimators of the earnings equations are also valid out of the supports of individual characteristics for which they were estimated. Empirical evidence will suggest that this assumption tends to overestimate the component of the gap attributable to differences in the rewards.

Along with the misspecification problem associated with gender differences in the supports, the BO decomposition is only informative about the average unexplained difference in wages. It is therefore not capable of addressing the distribution of these unexplained differences.

In this paper I use matching to highlight the problem of gender differences in the supports and provide information about the distribution of the unexplained pay differences. The approach does not require any estimation of earnings equations and hence no validity-out-of-the-support assumptions. Through the use of matching I propose a new decomposition of the wage gap that accounts for differences in the distribution of individuals’ characteristics. The methodology is implemented using data from Peru between the years of 1986 and 1999.

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1 Originally pointed out by Rubin (1977) and recently emphasized by Heckman, Ichimura, and Todd (1997).

2 For a discussion about typically female-dominated occupations and occupational segregation by gender in Latin America during the 1990s see Deutsch, et al. (2002).
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II. Relevant Matching and Wage Gap Decompositions Literature

Matching techniques, concerned with the comparison of groups with similar characteristics, have been of special interest to experimental design and statistics. However, it was not until the introduction of propensity scores in experimental designs by Rosenbaum and Rubin (1983) that the matching subject entered the discussion of the estimation of causal effects. As a result, a debate started in the economic literature about the widespread use of matching not only in experimental but also in nonexperimental designs. The matching framework is useful to address the statistical problem of analyzing the impact of a treatment on a population. For that purpose, one compares groups of observations with similar observable characteristics (or combinations of them), but different treatments. Under different sets of identifying assumptions, different estimators can be used to measure the impact of treatment.

Blinder (1973) and Oaxaca (1973) proposed a methodology to decompose wage gaps in terms of explained and unexplained components. The method is based on the separate estimation of earnings equations for the two groups being compared, namely females and males: \( \hat{y}F = \hat{\beta}F \hat{x}F \) and \( \hat{y}M = \hat{\beta}M \hat{x}M \). Thus, the wage gap can be expressed as \( \hat{y}M - \hat{y}F = \hat{\beta}M \hat{x}M - \hat{\beta}F \hat{x}F \). Then, the method requires the addition and subtraction of the term \( \hat{\beta}F \hat{x}M \) (or alternatively, \( \hat{\beta}M \hat{x}F \)), which can be interpreted as the counterfactual situation, “What would the earnings for a male (female) with average individual characteristics be, in the case that he (she) is rewarded for his (her) characteristics in the same way as the average female (male) is rewarded?” After algebraic manipulation, the wage gap takes the form \( \tilde{y}M - \tilde{y}F = \hat{\beta}F (\tilde{x}M - \tilde{x}F) + (\hat{\beta}M - \hat{\beta}F) \hat{x}M \). The first component of the right-hand side, \( \hat{\beta}F (\tilde{x}M - \tilde{x}F) \), is attributed to differences in average characteristics between males and females, while the second component \( (\hat{\beta}M - \hat{\beta}F) \hat{x}M \), is attributed to differences in average rewards to individuals’ characteristics. Juhn, Murphy, and Pierce (1993) extended the “characteristics-rewards” decomposition into one that considers three components: observable characteristics, observable rewards, and unobserved heterogeneity.

Dolton and Makepeace (1987) and Munroe (1988) pointed out a limitation of the original BO approach. The wage gap decomposition is informative only with regard to the average unexplained pay differences but not its distribution. One strategy for overcoming that distribution limitation, proposed by Buchinsky (1994), has been the estimation of quintile earnings equations. Another strategy, proposed by Jenkins (1994) and Hansen and Wahlberg (1999), used Generalized Lorenz Curves (GLC) for both observed earnings and predicted counterfactual earnings. These strategies ignore the problem of gender differences in the supports that this paper addresses.

The idea of extending the BO decomposition to a semiparametric setup, in order to explore the distribution of unexplained differences, can be found in DiNardo, Fortin, and Lemieux (1996). They estimated earnings equations nonparametrically by means of kernel estimations, facing the “curse of dimensionality” that arises when there are many explanatory variables in nonparametric setups. Another related semiparametric approach was proposed by Donald, Green, and Paarsch (2000). By adapting techniques from the duration literature to the estimation of density functions, they explored differences in wage distributions between Canada and the United States. Similarly, Bourguignon, Ferreira, and Leite (2002) discussed differences in density functions by adapting tools from the microsimulation literature to generate sequences of counterfactual densities and compare earnings distributions in Mexico, Brazil, and the United States.

A setup closely related to the one I use in this paper is proposed by Barsky et al. (2002). In their paper, Barsky et al. decomposed the black-white wealth gap in the United States based exclusively on income, avoiding in this way the dimensionality problem. Also, Black et al. (2004) analyzed wage gaps among the highly educated in the United States. Both studies recognized the importance of differences in the supports and restricted the comparison to the common support. Pratap and Quintin (2002), also using a propensity-scores matching approach, measured wage differences between the formal and informal sectors in Argentina.

My approach differs from that of Pratap and Quintin (2002) in two ways: the explicit assumption that the supports of the distributions of individual characteristics are different—which for the case of gender differences matters substantially—and the use of matching on characteristics instead of propensity scores. Furthermore, to address the issue also raised by Barsky et al. (2002) and Black et al. (2004). I propose a decomposition that also accounts for the out-of-common-support observations while measuring wage gaps in Peru.

III. The Link between Matching and Wage Gap Decompositions in a Nonparametric Setup

A. A Wage Gap Decomposition that Accounts for Differences in the Supports

Let \( Y \) denote the random variable that models individuals’ earnings and \( X \), the \( n \)-dimensional vector of individuals’ characteristics (such as age, education, and marital status) that are presumably related to earnings. Let \( F^M(\cdot) \) and \( F^F(\cdot) \) denote the conditional cumulative distribution functions of individuals’ characteristics \( X \), conditional on being male and female respectively, and \( dF^M(\cdot) \) and \( dF^F(\cdot) \) denote their corresponding probability measures. For a correct definition of the measures and integrals it is enough to assume that

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Since the measures on their respective partitioned domains, as is shown below.

The relationship governing these random variables is modeled by the functions $g^M (\cdot)$ and $g^F (\cdot)$, representing the expected value of earnings, conditional on characteristics and gender: $E[Y|M, X] = g^M (X)$ and $E[Y|F, X] = g^F (X)$.

It follows that

$$E[Y|M] = \int_{SM} g^M(x) dF^M(x)$$

and

$$E[Y|F] = \int_{SF} g^F(x) dF^F(x),$$

where $SM$ denotes the support of the distribution of characteristics for males and $SF$ the support of the distribution of characteristics for females. In such a way, the wage gap, defined as

$$\Delta = E[Y|M] - E[Y|F],$$

can be expressed as

$$\Delta = \int_{SM} g^M(x) dF^M(x) - \int_{SF} g^F(x) dF^F(x). \quad (1)$$

Considering that the support of the distribution of characteristics for females, $SF$, is different from the support of the distribution of characteristics for males, $SM$, each integral is split over its respective domain into two parts: within the intersection and out of the common support.

$$\Delta = \left[ \int_{SM \cap SF} g^M(x) dF^M(x) + \int_{SM \cap SF} g^M(x) dF^M(x) \right]$$

$$- \left[ \int_{SM \cap SF} g^F(x) dF^F(x) + \int_{SF \cap SM} g^F(x) dF^F(x) \right].$$

Since the measures $dF^F (\cdot)$ and $dF^M (\cdot)$ are identically zero out of their respective supports (by definition), the domains for the first and fourth integrals (the "noncommon support" integrals) can be extended to $\overline{SF}$ and $\overline{SM}$ respectively, without affecting their corresponding values. Also, every integral can be adequately rescaled in order to obtain expressions involving expected values of $g^F (X)$ and $g^M (X)$, conditional on their respective partitioned domains, as is shown below.

$$\Delta = \left[ \int_{SM} g^M(x) dF^M(x) \mu^M(S^F) \right]$$

$$+ \left[ \int_{SM \cap SF} g^M(x) dF^M(x) \mu^M(S^F) \right]$$

$$- \left[ \int_{SM \cap SF} g^F(x) dF^F(x) \mu^F(S^M) \right]$$

$$- \left[ \int_{SF \cap SM} g^F(x) dF^F(x) \mu^F(S^M) \right].$$

Now, replacing $\mu^F (SM)$ by $1 - \mu^F (\overline{SM})$ and $\mu^M (SF)$ by $1 - \mu^M (\overline{SM})$, the gap decomposition can be expressed in the following way:

$$\Delta = \left[ \int_{SM} g^M(x) dF^M(x) \mu^M(S^F) - \int_{SM} g^M(x) dF^M(x) \mu^M(S^F) \right]$$

$$+ \left[ \int_{SM \cap SF} g^M(x) dF^M(x) - \int_{SM \cap SF} g^M(x) dF^M(x) \right]$$

$$+ \left[ \int_{SF \cap SM} g^F(x) dF^F(x) - \int_{SF \cap SM} g^F(x) dF^F(x) \right].$$

Finally, the second pair of integrals in this expression (those that are computed over the common support, $SM \cap SF$) can be decomposed further by adding and subtracting the element that permits them to evaluate the counterfactual mentioned above, $\int_{SM} g^M(x) \frac{dF^M}{d\mu^M} (x)$ (I use $\frac{dF^M}{d\mu^M}$ to denote the measure with signal induced by the original measures $dF^M$ and $dF^F$ and the corresponding arithmetic operations), obtaining

$$\Delta = \left[ \int_{SM} g^M(x) dF^M(x) \mu^M(S^F) - \int_{SM} g^M(x) dF^M(x) \mu^M(S^F) \right]$$

$$+ \left[ \int_{SM \cap SF} g^M(x) \frac{dF^M}{d\mu^M}(S^F)(x) \right]$$

$$+ \left[ \int_{SM \cap SF} g^M(x) - g^F(x) \frac{dF^F}{d\mu^F}(S^M)(x) \right]$$

$$+ \left[ \int_{SF \cap SM} g^F(x) \frac{dF^F}{d\mu^F}(S^M)(x) - \int_{SF \cap SM} g^F(x) \frac{dF^F}{d\mu^F}(S^M)(x) \right].$$

\footnote{This is a generalization of the linear model in which $E[Y|X] = \beta X,$ where $\beta$ is a $1 \times n$ parameter vector and $X$ is an $n \times 1$ regressor vector, which I denote by...
\[ \Delta = \Delta_M + \Delta_x + \Delta_0 + \Delta_\phi. \]  
(2)

The typical interpretation of the wage gap decomposition applies, but only over the common support. In this new construction, two new additive components have been included, resulting in a four-element decomposition.

The first component.

\[ \Delta_M = \left[ \int_{S_M} g^M(x) \frac{dF^M(x)}{\mu^M(S')} - \int_{S_F} g^M(x) \frac{dF^F(x)}{\mu^F(S')} \right] \mu^M(S') \],

(3)

is the part of the gap that can be explained by differences between two groups of males—those who have characteristics that can be matched to female characteristics and those who do not. This component accounts for that part of the gap that would disappear in the event that there are no males with combinations of characteristics that remain entirely unmatched by females. Alternatively, this component would also disappear if those males with individual characteristics that are not matched by females are paid, on average, as much as matched males. It is computed as the difference between the expected male wages out of the common support minus the expected male wages in the common support, weighted by the probability measure (under the distribution of characteristics of males) of the set of characteristics that females do not reach.

The second component,

\[ \Delta_x = \left[ \int_{S_{M*}\cap S_F} g^M(x) \left[ \frac{dF^M(x)}{\mu^M(S')} - \frac{dF^F(x)}{\mu^F(S')} \right] \right] (x), \]

(4)

is the part of the wage gap that can be explained by differences in the distribution of characteristics of males and females over the common support. In the linear BO setup this corresponds to the component \( \hat{\beta}^M \cdot (\hat{\xi}^M - \hat{\xi}^F) \).

The third component,

\[ \Delta_0 = \left[ \int_{S_{M*}\cap S_F} \left[ g^M(x) - g^F(x) \right] \frac{dF^F(x)}{\mu^F(S')} \right] \]

(5)

corresponds to the “unexplained part.” It is the share of the wage gap that cannot be attributed to differences in characteristics of the individuals and is typically attributed to a combination of both unobservable characteristics and discrimination. In the linear Blinder-Oaxaca setup this corresponds to the component \( (\hat{\beta}^M - \hat{\beta}^F) \cdot \hat{x}^F \).

The fourth component,

\[ \Delta_\phi = \left[ \int_{S_F} g^F(x) \frac{dF^F(x)}{\mu^F(S')} - \int_{S_F} g^F(x) \frac{dF^F(x)}{\mu^F(S')} \right] \mu^F(S'), \]

(6)

is the part of the gap that can be explained by the differences in characteristics between two groups of females, those who have characteristics that can be matched to male characteristics and those who do not. It accounts for the part of the gap that would disappear should it ever be the case that all females would have characteristics that can be matched to the population of males. It would also disappear if unmatched females are paid, on average, as much as matched females. It is computed as the difference between the expected female wages, in and out of the common support, weighted by the probability measure (under the distribution of characteristics of females) of the set of characteristics that males do not reach.

The wage gap has been broken into four additive components. Three of them can be attributed to the existence of differences in individuals’ characteristics (\( \Delta_M, \Delta_x, \) and \( \Delta_\phi \)), and the fourth (\( \Delta_0 \)) to the existence of a combination of differences in unobservable characteristics and discrimination. In that sense, the wage gap can be expressed as

\[ \Delta = (\Delta_M + \Delta_x + \Delta_\phi) + \Delta_0. \]

(7)

Then, it can be interpreted as it is traditionally done in the linear BO setup, with two components: one that can be explained by differences in observable characteristics and one that cannot.

Under this framework, I will introduce the matching procedure in order to estimate these four components. I will resample all females without replacement and match each observation to one synthetic male, with the same observable characteristics and with a wage obtained from averaging all males with exactly the same characteristics \( x \). For the scope of this paper, I consider only characteristics that can be described with discrete variables and perfect matching. The matching algorithm in its basic form can be summarized as follows:

- Step 1: Select one female from the sample (without replacement).
- Step 2: Select all the males that have the same characteristics \( x \) as the female previously selected.\(^5\)
- Step 3: With all the individuals selected in step 2, construct a synthetic individual whose wage is the average of all of them and match him to the original female.
- Step 4: Put the observations of both individuals (the synthetic male and the female) in their respective new samples of matched individuals.
- Repeat steps 1 through 4 until exhausting the original female sample.

\(^5\) The condition of perfect matching and discrete domain for the characteristics could be relaxed to some extent. That would require the introduction of some notion of distance and a tolerance for the maximum distance acceptable for two sets of characteristics to be considered as “matchable.” For the sake of simplicity and in order to convey the main message of this paper we abstract from that here.
As a result of the application of this one-to-many matching, I generate a partition of the data set. The new data set contains observations of matched females, matched males, unmatched females, and unmatched males, where the sets of matched males and females have the same empirical distributions of probabilities for characteristics \( X \).

The purpose of resampling without replacement from the sample of females and with replacement from the sample of males is to preserve the empirical distribution of characteristics for females (being the case that the support for that distribution is finite). The generation of a counterfactual—continuing the analogy with the original Blinder-Oaxaca setup—can also be done the opposite way. It can be generated by resampling without replacement for males and with replacement for females with the appropriate changes in the interpretation of the four components derived.

In such a way, the estimation of the four components previously presented is reduced to computations of conditional expectations and empirical probabilities without the need to estimate the nonparametric earnings equations \( g^M(\cdot) \) and \( g^F(\cdot) \).

\[
\Delta_\mu = \mu^M(\text{Unmatched})(E_{M,\text{unmatched}}[Y|M] - E_{M,\text{matched}}[Y|M]),
\]

\[
\Delta_\nu = E_{M,\text{matched}}[Y|M] - E_{F,\text{matched}}[Y|M],
\]

\[
\Delta_0 = E_{F,\text{matched}}[Y|M] - E_{F,\text{unmatched}}[Y|F],
\]

\[
\Delta_\rho = \mu^F(\text{Unmatched})(E_{F,\text{matched}}[Y|F] - E_{F,\text{unmatched}}[Y|F]).
\]

The use of this matching criterion avoids any type of parametric assumptions on the random variables involved in the analysis (or combinations of them). It is solely based on the modeling assumption that individuals with the same observable characteristics should be paid the same regardless of their gender.

The analysis presented here raises a point to be taken into account in the traditional setup of the BO decomposition, one that has not received considerable attention but plays an important role: the supports of the distributions of characteristics for females and males may not overlap completely, thus it is necessary to restrict the decomposition in terms of differences in characteristics and differences in coefficients to the common support, where the wages comparison is appropriate. Using the BO decomposition, it is necessary to implicitly make out-of-the-support assumptions on the linear estimators obtained by the regressions.\footnote{Namely, the assumption that the fitted regression can be extended for combinations of individual characteristics that have not been found empirically in the data sets.} assumptions that may seem plausible, but for which it is impossible to find evidence in favor or against. With the decomposition proposed here it is not necessary to make these kinds of assumptions. Moreover, I propose a way to compute those components of the gap that correspond to the nonoverlapping supports (\( \Delta_\mu \) and \( \Delta_\rho \)).

As will be shown, it is an empirical regularity that the unmatched males have average wages above the average wages of their matched peers. Hence, running regressions to estimate earnings equations for all males without recognizing this empirical regularity tends to overestimate the unexplained component (\( \Delta_0 \)) in the BO decomposition.

B. Computing Standard Errors for the Average Unexplained Gender Pay Differences

One of the advantages of using matching is that it generates an empirical distribution of unexplained gender pay differences. It is only necessary to compute the differences in hourly wages between matched females and males, averaging all the differences. In this section, I will use this trait and compute standard errors for the average unexplained gender pay differences.

Specifically, the unexplained pay differences component, \( \Delta_0 \), can be expressed as

\[
\Delta_0 = \int_{S^M \cup S^F} g^M(x) \frac{dF(x)}{\mu^F(S^M)} - \int_{S^M \cup S^F} g^F(x) \frac{dF(x)}{\mu^F(S^M)}.
\]

Let \( n_F(x) \) denote the number of females in the matched sample who report the set of characteristics \( x \); similarly, \( n_M(x) \) denotes the number of males reporting such combination of characteristics. Also, \( n_F \) and \( n_M \) denote the total number of females and males in the matched sample respectively.

Using \( g^M(x) = \frac{1}{n_M(x)} \sum_{y_M \in S^M} y_M^M \) and \( g^F(x) = \frac{1}{n_F(x)} \sum_{y_F \in S^F} y_F^F \), I construct the sample analog

\[
\delta_0 = \sum_x \left( \sum_{y_M \in S^M} \frac{1}{n_M(x)} y_M^M \right) \frac{n_F(x)}{n_F} - \sum_{j=1}^{n_F} y_F^F \frac{1}{n_F}
\]

which, after denoting \( \sum_{y_M \in S^M} \frac{1}{n_M(x)} y_M^M \) by \( \tilde{y}_M^M(x) \) (the sample average of earnings for males that exhibit the set of characteristics \( x \)) and \( \sum_{j=1}^{n_F} y_F^F \) by \( \tilde{\omega}_F^F \) (the sample proportion of females that exhibit the set of characteristics \( x \)) can be expressed as

\[
\delta_0 = \sum_x \tilde{y}_M^M(x) \tilde{\omega}_F^F - \sum_{j=1}^{n_F} y_F^F \frac{1}{n_F}.
\]
From this expression, the asymptotic distribution of the second component of the right-hand side of equation (9) is 

\[ \sqrt{n_F}(\hat{y}^F - y^F) \xrightarrow{n_F \to \infty} N(0, \sigma_2^2). \]

The asymptotic distribution of the first component of the right-hand side of equation (9) can be computed applying the delta method.

Let \( K \) denote the number of values that the set of characteristics \( x \) can attain, \( \hat{W}^F \), the \( K \)-dimensional vector whose elements are the weights \( \hat{\omega}^F(x_i) (i = 1, \ldots, K) \), and \( \bar{Y}^M \), the \( K \)-dimensional vector whose elements are the conditional means \( \bar{y}^M(x_i) \). Finally, let \( f_i \) denote the population proportion of females that exhibit the set of characteristics \( x_i (i = 1, \ldots, K) \) and let \( \sigma_i^2 \) denote the population variance of male wages exhibiting the set of characteristics \( x_i (i = 1, \ldots, K) \). Using this notation, the first component of the right-hand side of equation (9) can be expressed as \( \hat{P} = \hat{W}^F \cdot \bar{Y}^M \).

The asymptotic distributions of \( \hat{W}^F \) and \( \bar{Y}^M \) are respectively

\[ \sqrt{n_F}(\hat{W}^F - W^F) \xrightarrow{n_F \to \infty} N(0, V_{\hat{w}^F}) \] and
\[ \sqrt{n_M}(\bar{Y}^M - Y^M) \xrightarrow{n_M \to \infty} N(0, V_{\bar{y}^M}), \]

where

\[ V_{\hat{w}^F} = \begin{bmatrix} f_1(1 - f_1) & -f_1f_2 & \ldots & -f_1f_K \\ -f_2f_1 & f_2(1 - f_2) & \ldots & -f_2f_K \\ \vdots & \vdots & \ddots & \vdots \\ -f_Kf_1 & -f_Kf_2 & \ldots & f_K(1 - f_K) \end{bmatrix} \]

and

\[ V_{\bar{y}^M} = \begin{bmatrix} \sigma_1^2 & 0 & \ldots & 0 \\ 0 & \sigma_2^2 & 0 & \ldots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_K^2 \end{bmatrix}. \]

Assuming that \( n_F/n_M \to \alpha \in (0, 1) \) for some \( \alpha > 0 \) (large enough) and that the sample of females is independent of the sample of males, then

\[ \sqrt{n_M}(\hat{W}^F - W^F) \xrightarrow{n_M \to \infty} N(0, V), \]

where

\[ V = \begin{bmatrix} V_{\hat{w}^F} & 0 \\ 0 & V_{\bar{y}^M} \end{bmatrix}. \]

Applying the delta method (\( \frac{\hat{p}^F}{\sqrt{n_F}} = Y^M \) and \( \frac{\hat{p}^M}{\sqrt{n_M}} = W^F \)), the limiting distribution of the product \( \hat{P} \) can be approximated as

\[ \sqrt{n_M}(\hat{P} - P) \sim N \left( 0, \left[ \begin{array}{c} \bar{y}^M W^F \end{array} \right] V \left[ \begin{array}{c} \bar{y}^M \end{array} \right] \right). \]

In such a way, the asymptotic variance of the first component of the right-hand side of equation (9) can be computed as

\[ \sum_{i=1}^{K} \frac{\hat{\omega}^F(x_i)(1 - \hat{\omega}^F(x_i))}{\alpha^2} (\bar{y}^M(x_i))^2 + \sigma_i^2(\hat{\omega}^F(x_i))^2 \]

\[ -2 \sum_{i=1}^{K} \sum_{j=1}^{i-1} \frac{\hat{\omega}^F(x_i)\hat{\omega}^F(x_j)}{\alpha^2} \bar{y}^M(x_i) \bar{y}^M(x_j). \]

Applying the same delta method on restricted samples, according to different sets of characteristics \( x \), I also obtained estimators for the mean and the standard deviation of the unexplained differences in pay for those different sets of characteristics. The results will be shown in the next section.

### IV. Implementing the Decomposition: Peru 1986–1999

The data come from two Peruvian national surveys: the National Household Surveys (Encuestas Nacionales de Hogares) and the Specialized Employment Survey (Encuesta Especializada de Empleo) undertaken by the Peruvian Ministry of Labor and Social Promotion (MTPS) from 1986 to 1995 (not including 1988), and by the National Institute of Statistics and Informatics (INEI) from 1996 to 1999. For homogenizing purposes—and since almost one-half of the Peruvian labor force works in Lima—only workers 14 years or older in metropolitan Lima have been considered for this study. Peru is an interesting country to analyze with a matching tool, since the problem of the nonoverlapping supports matters more than in developed economies. The percentages of individuals that cannot be matched simultaneously on age, education, and marital status are around 26% for females and 29% for males. On the other hand, in the U.S. labor market, the percentage of females and males that cannot be matched on the same variables reaches only 8%. Table 1 reports averages for females and males on the main observable characteristics for the Peruvian labor market during the period of analysis.

| Table 1.—Average Observable Characteristics by Gender, Peru 1986–1999 |
|-----------------------------|-----------------------------|
|                            | Females                     | Males                      |
| Age                         | 36.45 (13.45)               | 34.05 (12.38)              |
| Education                   | 10.84 (3.76)                | 9.95 (4.33)                |
| Migratory condition (1 = migrant) | 0.5456 (0.4979)         | 0.5602 (0.4964)            |
| Marital status (1 = married) | 0.6095 (0.4879)            | 0.4351 (0.4958)            |
| Job formality (1 = formal)   | 0.4526 (0.4978)            | 0.3720 (0.4834)            |
| Union membership (1 = unionized) | 0.1671 (0.3731)         | 0.1098 (0.3127)            |
| Tenure (years)              | 7.19 (6.69)                 | 5.02 (6.91)                |
| Hours worked per week       | 53.00 (19.78)               | 46.00 (23.30)              |

Standard deviations in parentheses.

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8 Two individuals are matched only if they were born in the same year, completed the same number of years of schooling, have the same marital status (currently single versus currently married) and have the same migratory condition (born in Lima versus born out of Lima).
in the paper, $\Delta = 45\%$). A decomposition of such a wage gap, after matching on age, education, marital status, and migratory condition, reveals that 11% is accounted for by differences in the supports ($\Delta_F = 2\%$ and $\Delta_M = 9\%$). Gender differences in the distributions of individual characteristics on the common support explain 6% of average females’ wages (that is, $\Delta_x = 6\%$). Finally, 28% remains unexplained ($\Delta_0 = 28\%$).

The extent to which the raw gender wage gap ($\Delta$) can be explained depends on the number of explanatory variables used in the matching procedure. However, the likelihood of matching decreases with the number of explanatory variables as well. In table 2, I show the unexplained gender wage gap ($\Delta_0$) as well as the percentages of females and males that remain unmatched for different combinations of age, education, marital status, and migratory condition. Next, in table 3, I show the mean and standard errors for the unexplained gender pay differences for the whole population and selected groups.

The unexplained gender wage gap among single individuals does not differ substantially from that among the married.

### Table 2.—Unexplained Gender Wage Gap and Nonoverlapping Supports, Peru 1986–1999

<table>
<thead>
<tr>
<th>Variables</th>
<th>Wage Gap as % of Female Average Wages</th>
<th>% of Females Unmatched</th>
<th>% of Males Unmatched</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age, education, marital status, and migratory condition</td>
<td>28%</td>
<td>35%</td>
<td>40%</td>
</tr>
<tr>
<td>Age, education, and marital status</td>
<td>24%</td>
<td>26%</td>
<td>29%</td>
</tr>
<tr>
<td>Age, education, and migratory condition</td>
<td>29%</td>
<td>23%</td>
<td>30%</td>
</tr>
<tr>
<td>Education, marital status, and migratory Condition</td>
<td>31%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>Age, marital status, and migratory Condition</td>
<td>39%</td>
<td>3%</td>
<td>4%</td>
</tr>
<tr>
<td>Age and education</td>
<td>27%</td>
<td>16%</td>
<td>20%</td>
</tr>
<tr>
<td>Age and marital status</td>
<td>30%</td>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td>Age and migratory condition</td>
<td>40%</td>
<td>0%</td>
<td>2%</td>
</tr>
<tr>
<td>Education and migratory condition</td>
<td>31%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Education and marital status</td>
<td>38%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Marital status and migratory condition</td>
<td>40%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Age</td>
<td>40%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Education</td>
<td>38%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Marital status</td>
<td>39%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Migratory condition</td>
<td>47%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

### Table 3.—Unexplained Gender Wage Gap for Selected Groups: Means and Standard Errors, Peru 1986–1999

| (After controlling for age, education, marital status, and migratory condition) |
|-------------------------------------|------------------|------------------|
| Mean                               | Std. Error       |
| All                                 | 0.2803           | 0.0189           |
| Marital status                      |                  |                  |
| Single                              | 0.2751           | 0.0242           |
| Married                             | 0.2862           | 0.0289           |
| Migratory condition                 |                  |                  |
| Born in Lima                        | 0.3067           | 0.0300           |
| Not born in Lima                    | 0.2840           | 0.0269           |
| Education                           |                  |                  |
| Singles                             |                  |                  |
| No education                        | 0.3246           | 0.1203           |
| High school diploma                 | 0.1385           | 0.0301           |
| College degree                      | 0.5740           | 0.1026           |
| Married                             |                  |                  |
| No education                        | 0.2352           | 0.0532           |
| High school diploma                 | −0.0143          | 0.0329           |
| College degree                      | 1.0701           | 0.1092           |

The gap among migrants (not born in Lima) is slightly higher than among the locals, but the magnitude of the standard errors makes such difference not significant. The gender wage gap by educational level shows more heterogeneity. There are no significant gender pay differences among married individuals with a high school diploma, however the gap reaches 107% among those who are married and obtained a college degree. Next, I analyze the distribution of the unexplained gender pay differences, using the advantages of matching over the traditional parametric approach.

### B. Differences in Hourly Wages between Matched Samples

The following analysis is based on the distribution of wages for males and females using the cumulative (empirical) distribution function of hourly wages for the matched samples of females and males. Figure 1 shows that when plotting the cumulative distribution functions, the “wages for females” random variable is stochastically dominated by the matched version of the “wages for males” random variable. The cumulative functions of wages for all females and for matched females are similar and they overlap in the figure. On the contrary, the corresponding cumulative func-
tions for males differ. The one that stocastically dominates
the rest of them (located at the most southeastern area of the
figure) is the original distribution for males. The distribution
function for matched males gets closer to the functions for
females. These results are in accordance with the fact that
the resampling has been done without replacement for
females and with replacement among males. As a result of
these replacement/no-replacement decisions, the cumulative
function for females was not expected to change, while the
cumulative function for males was. In fact, the cumulative
function for matched males gets closer to the cumulative
function for females.

Figure 1 inspires a quantile analysis in the following way:
at any height (percentile), the horizontal distance between
the two cumulative distribution functions obtained after
matching is a measure of the unexplained gender wage gap
at the respective percentile. Figure 2 shows, by percentiles,
these measures of the gender wage gap that remain after
matching. The plot shows that for the first 90 percentiles of
the distribution of hourly wages for males and females there
are no major differences in hourly wages. The gap is roughly
below 0.2 times the average wage for females. It is in the top
10% of the distributions of hourly wages for males and
females that the highest differences are found. At the 99th
percentile the gap attains a maximum of 2.2 times the
average wage of females. The plot shows evidence that the
gender pay differences in the bottom percentiles of the
distribution do not considerably contribute to the aggregate
measure of gender pay differences in Peru for the period of
analysis. The average gender wage gap in Peru is driven by
gender pay differences at the top percentiles of the wage
distribution.

However, the differences found in hourly wages within
the bottom percentiles are small in absolute terms but not in
relative terms. Despite the fact that a male at the bottom 10th
percentile earns 12% of the average female wage at the same
percentile, the difference is 60% of a female’s earnings. These
percentage differences in hourly wages by percentiles of the
wage distributions are shown in figure 3. The relative
gender wage gap by wage percentiles shows a slight U-
shape in which the minimum gap, 18%, is found among
those individuals whose wages are between the 8th and 9th
deciles. The maximum is found among the poor, 95%.

C. Matching versus Linear Regressions: An Empirical
Comparison

After the introduction of the matching approach which
decomposes the gender wage gap avoiding linear regressions,
there is a comparative question: to what extent do the
results obtained by matching differ from those obtained by
linear regressions? In some sense, matching is equivalent to
Blinder-Oaxaca when the estimations of the earnings equa-
tions for males and females are restricted to the common
support and performed with the same matching variables
and all their possible powers and interactions. We should
therefore expect similar results from both. Using data for
only 1999, I empirically show that there are no substantial
differences on the common support. However, failure to
recognize gender differences in the supports accounts for an
overestimation of the unexplained gender pay differences.

As was pointed out previously, the BO decomposition,
based on linear regressions, depends on the linear specifi-
cation of the earnings equations. For this purpose, I compare
the matching results with those obtained from four different
linear specifications. The first specification includes the
following variables: age (as a continuous variable), age
squared, three dummies measuring educational attainment
("elementary school," "high school," and "college degree,"
with "no education" as the base category) eleven dummies
measuring years of occupational experience, and all the
interactions between educational attainment and tenure. The
second uses the same dummies and interactions for educa-
tional attainment and occupational experience as in speci-
fication 1, but replaces the age and age squared variables
with 62 dummies. The third changes specification 2 by
replacing the three dummies for educational attainment with
a set of 21 dummies measuring years of schooling (and their
interactions with the dummies for years of experience). The
fourth specification replaces the interactions between
schooling and occupational experience with the interactions
of age and occupational experience.
The use of dummies instead of continuous variables has two purposes: on the one hand this helps to identify gender differences in the supports and on the other, this reduces the dependence on the functional form of the earnings equations (as is the case for matching). Also, this comparative exercise considers two types of BO decompositions for the linear specifications: one without recognizing the existence of such gender differences in the supports and another that recognizes those differences and computes the decomposition only on the common support. Table 4 shows the results.

When comparing the two sets of linear specifications I find that differences in supports account for a large share of the gap, particularly because there is a substantial percentage of males with high wages and individual characteristics that have no female counterparts ($\Delta_0$). Therefore, accounting for gender differences in the supports changes the unexplained differences in earnings from an estimated average of 22.4%–27.7% to an estimate of 20.9%–22.9% for 1999. The previous comparisons—made for the same linear specifications but with different assumptions on the supports—show empirically that the failure to recognize gender differences in the supports implies a slight overestimation of the unexplained component of the gap ($\Delta_0$).

Comparing the decomposition based on linear regressions with the decomposition based on matching, I find no substantial differences. The unexplained wage gap obtained by matching (22.8%) falls inside the range of estimators obtained with the different linear specifications. Restricting the earnings equations to specific functional forms seems to have no substantial effect on the measure of unexplained differences, as long as the regressions are estimated only on the common support. It should be noted that in measuring the gender wage gap in per-hour terms (and only between working males and females), I do not take into consideration the effects of participation in the labor market.

### V. Conclusions

This paper introduces a nonparametric technique to decompose gaps in terms of explained and unexplained components, carefully considering the problem of gender differences in the supports. Indeed, one of the purposes of the paper was to challenge the linear specifications that involve the estimation of earnings equations and to propose matching as an alternative for a world in which the relationships that govern the comovement of wages and individual characteristics are not necessarily linear. In that respect, I have empirically found that there are no substantial differences that matter for the wage gap decomposition. In the end, the linearity assumption does not influence the wage gap decomposition, provided that it is estimated only over the common support.

There is a noteworthy issue that this matching approach raises: the importance of recognizing the gender differences in the supports. That is, “not all males are comparable to all females.” The failure to recognize this problem implies an overestimation of the unexplained component of the wage gap. Also, there is an important share of the wage gap that can be attributed to the fact that a significant number of males exhibit a set of characteristics that have no female counterparts, and these characteristics are highly rewarded in the labor markets.

Besides the issue of gender differences in the supports, matching also has an advantage over the traditional linear regressions. By means of matching, instead of obtaining only average unexplained pay differences, it is possible to obtain a distribution for such measures. To explore the distribution of unexplained pay differences provided insightful results. The average gender wage gap is mainly driven by gender pay differences at the top percentiles of the wages distributions. Wages at the highest quintile of the wage distribution for females and males explain more than one-half of the average wage gap in Peru for the period of analysis. At the poorest percentiles of earnings, the wage gap in absolute terms is small and does not contribute substantially to the average wage gap of the population: but the same wage gap in the poorest percentiles, measured in relative terms, is the highest among all percentiles (around 94%). Also, I find that there is more dispersion of the
unexplained gender pay differences among the married than the single. In addition, there is evidence of a substantially higher (and more disperse) unexplained gender wage gap among the highly educated (more than a college degree).

In general, this matching approach can also be used to control for observed characteristics in other measures for which it is expected to find some sort of explained and unexplained components. Its application is not confined to the gender wage gap literature. Its use only requires the generation of the adequate counterfactuals. However, the advantages of this form of matching over linear regressions are not a free lunch. The two most important costs are the need to use only discrete variables and the curse of dimensionality, which in some applications may constitute non-negligible elements. The inclusion of many explanatory variables—that is, the use of many matching characteristics—or a high number of possible values for a single matching (discrete) variable may reduce the chances of obtaining an adequate number of matched observations, limiting as a consequence the usefulness of this tool. Nonetheless, the issue about the nonoverlapping supports of observable characteristics—one that is regularly overlooked and seems to bias upwardly the estimators of unexplained gaps—is made apparent only thanks to the use of this form of matching.

REFERENCES